Automatic exploration of reduced floating-point representations in iterative methods

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Context

- Approximate computing use lower precision to reduce **time**, **communications' volume** and **energy** consumption
- Deep learning is driving the research in approximate computing
- Emergence of **new FP formats** that provide great speedup when combined with **custom hardware** (Google TPU [Jouppi et al., 2017], Intel's Nervanna Neural Network [Rao, 2018], IBM's SoC [Fleischer et al., 2018])

Objective: How can we harness these benefits for HPC?

State of the art

Reducing precision is a three-dimensional problem:

- **Spatial**: Finding code parts that can be rewritten into lower precision [Rubio-González et al., 2013, Lam et al., 2013, Graillat et al., 2016, Rubio-González et al., 2016]
- **Temporal**: Finding time lapses where working precision can be lowered. Works [Anzt et al., 2017, Haidar et al., 2018] based on *Iterative refinement* [Wilkinson, 1994]
- **Precision**: Finding the precision to use. Most of works focus on existing IEEE-754 formats (mixed-precision) but new formats exist (bfloat16 [LLC, 2019], DLfloat [dlf,])

Missing: a fine-grain exploration at bitwise level

Contributions

VPREC

- Simulating custom FP formats with correct rounding
- Easily testing new formats on HPC codes

Methodology to find lower precisions

- Automatic exploration on iterative scheme
- Gives lowest precision at bit level for each iteration
- Validation of the robustness to round-off errors

Validation on an industrial code (YALES2)

- Scale mixed-precision version on CRIANN cluster
- Communication's volume reduction and speedup

Example: Newton-Raphson method

k	x_{k+1}	s_k^{10}	s_k^2
0	0.0690266447076745	0.11	0.37
1	0.1230846130203958	0.21	0.70
2	0.1985746566605835	0.43	1.43
3	0.2732703639721015	0.84	2.79
4	0.3119369815109966	1.79	5.95
5	0.3181822938100336	3.40	11.3
6	0.3183098350392471	6.79	22.6
7	0.3183098861837825	13.6	45.2
8	0.3183098861837907	15.6	51.8
9	0.3183098861837907	15.6	51.8

```
double newton(double x0) {
  double x_k;
  double x_k1=x0;
  double b=PI;
  do {
    x_k  = x_k1;
    x_k1 = x_k*(2-b*x_k);
  } while (fabs((x_k1-x_k)/x_k)
                  >= 1e-15);
  return x_k1;
}
```

Table: (*left*) Convergence speed of Newton-Raphson for the computation of the inverse of π using the IEEE-754 `binary64` format (*right*). Highlighted digits in red are non significant

In the first iterations, most digits are incorrect, hinting that a low precision for the evaluation of f is enough

Question: How to find the minimal precision required to converge?

VPREC on the Newton-Raphson method

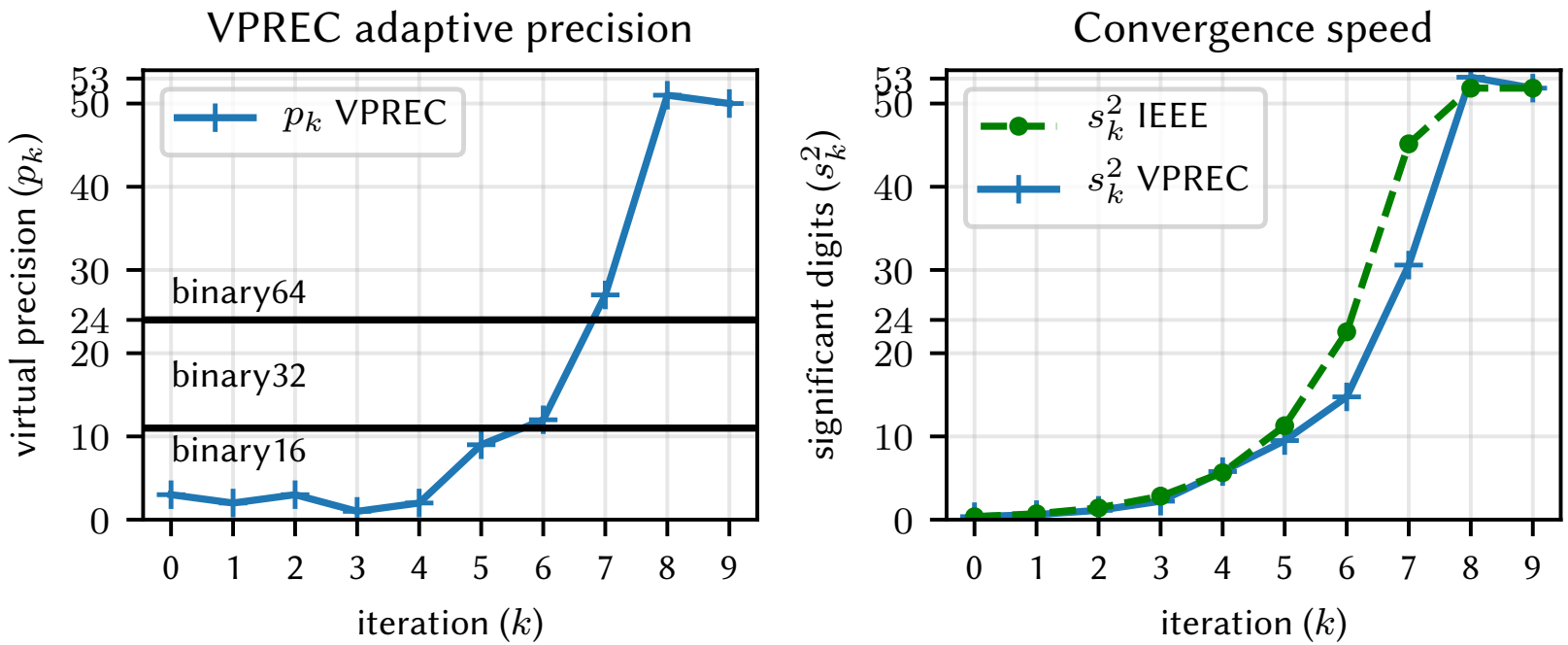
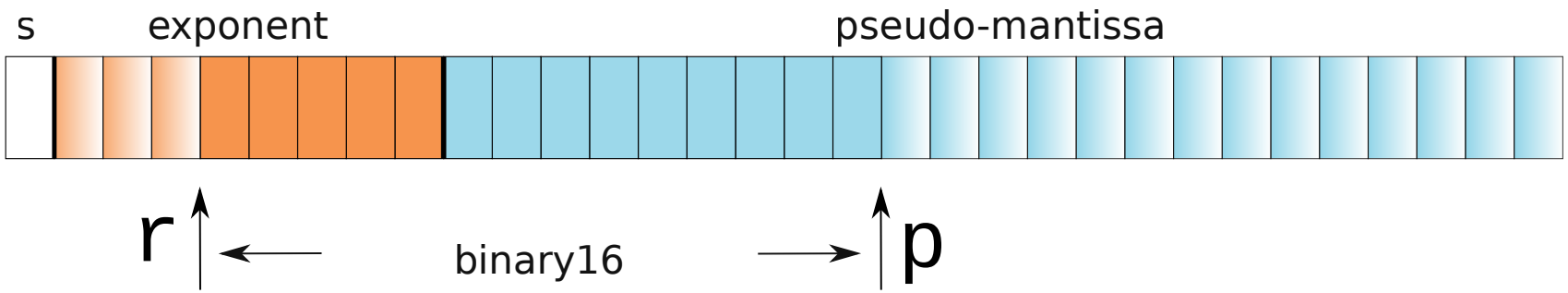


Figure: *(left)* Precision found with VPREC for Newton-Raphson. *(right)* Both the standard IEEE-754 binary64 version and the VPREC low precision configuration converge to 10^{-15} in nine iterations

Simulates any format that fits into a binary64

- Defined by couple (r, p), r=exponent size, p=pseudo-mantissa size
- Correctly *round to nearest ties to even*
- Handles `NaN`,`Inf` and denormals



By setting $r = 5$ and $p = 10$, VPREC simulates a *binary*16 embedded inside a *binary*32

VPREC rounding

Each FP operation $\bullet \in \{+, -, *, /\}$ is replaced by:

$$VPREC_{(r,p)}(x \bullet y) = \underbrace{\diamond_{(r,p)}}_{\text{OB}}(\underbrace{\diamond_{(r,p)}}_{\text{IB}}(x) \bullet \underbrace{\diamond_{(r,p)}}_{\text{IB}}(y))$$

where

$$\diamond_{(r,p)}(z) = \begin{cases} NaN_{b64} & \text{if } z = NaN_r \\ \pm\infty & \text{if } \lfloor \log |z| \rfloor > e_{max} \text{ or } z = \pm\infty \\ \pm 0 & \text{if } \lfloor \log |z| \rfloor < e_{min} - p \\ \circ_p(z) & \text{otherwise} \end{cases}$$

and

$$e_{max} = 2^{r-1} - 1, e_{min} = 1 - e_{max}$$

and

$\circ_p(x)$ is the *RoundToNearestTiesToEven* at precision p

Two modes : OutBound Precision (OB) or InBound Precision (IB)

VPREC rounding: example

Simulating `binary16` inside a `binary32`

- $r = 5, p = 10$ ($e_{max} = 15, e_{min} = -14$)
- $VPREC_{5,10}((2.903225 + 2.903225) * 256) =?$

$$
\begin{array}{rl}
 & 1.01110011100111001110000 \times 2^1 \\
+ & 1.01110011100111001110000 \times 2^1 \\
\hline
 & 1.01000010000100001000000 \\
\hline
\end{array}
$$

$$
\begin{array}{rl}
 & 1.01000010000000000000000 \times 2^2 \\
* & 1.00000000000000000000000 \times 2^8 \\
\hline
 & \\
\hline
\end{array}
$$

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* & 1.00000000000000000000000 \times 2^{8} \\
\hline
\diamond_{5,10} & 1.01000010000000000000000 \times 2^{16} \\
\hline
 & +Inf
\end{array}
$$

Verificarlo github.com/verificarlo/verificarlo

- Verificarlo is a numerical toolbox for the floating point model:
  - **Numerical validation**: detect numerical bugs and ensure numerical accuracy during code modernization
  - **Numerical optimization**: reduce precision, harness new floating point formats and architectures
- Replaces every FP operations by a call to Verificarlo interface
- Interface allows using various backends during execution [Denis et al., 2016, Chatelain et al., 2018]
- Operates on optimized code: evaluates the impact of compiler optimizations

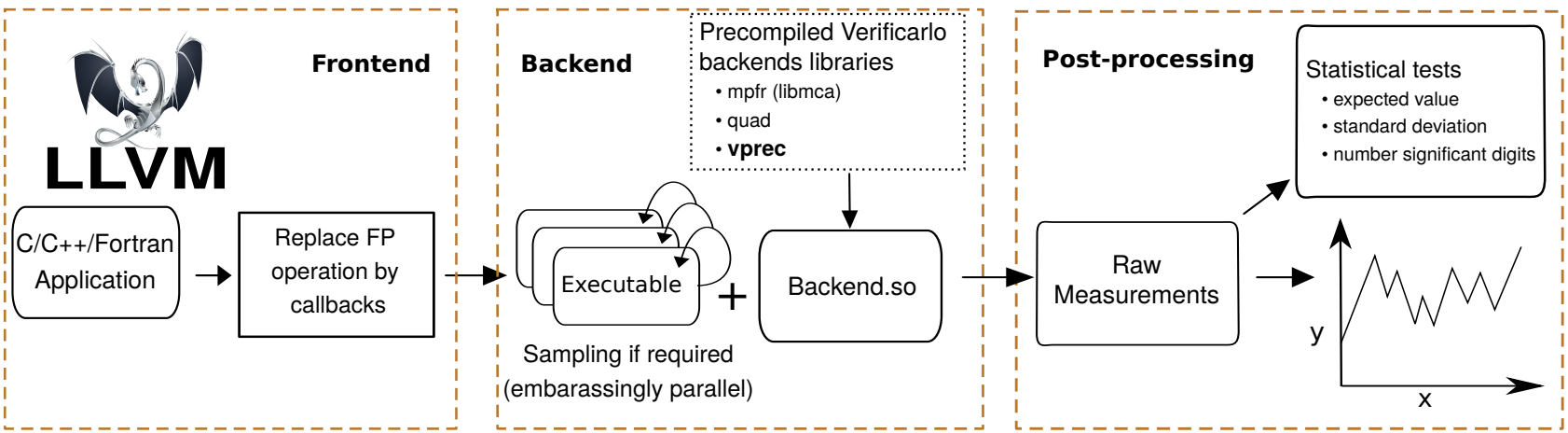


Figure: General Verificarlo exploration workflow with VPREC backend

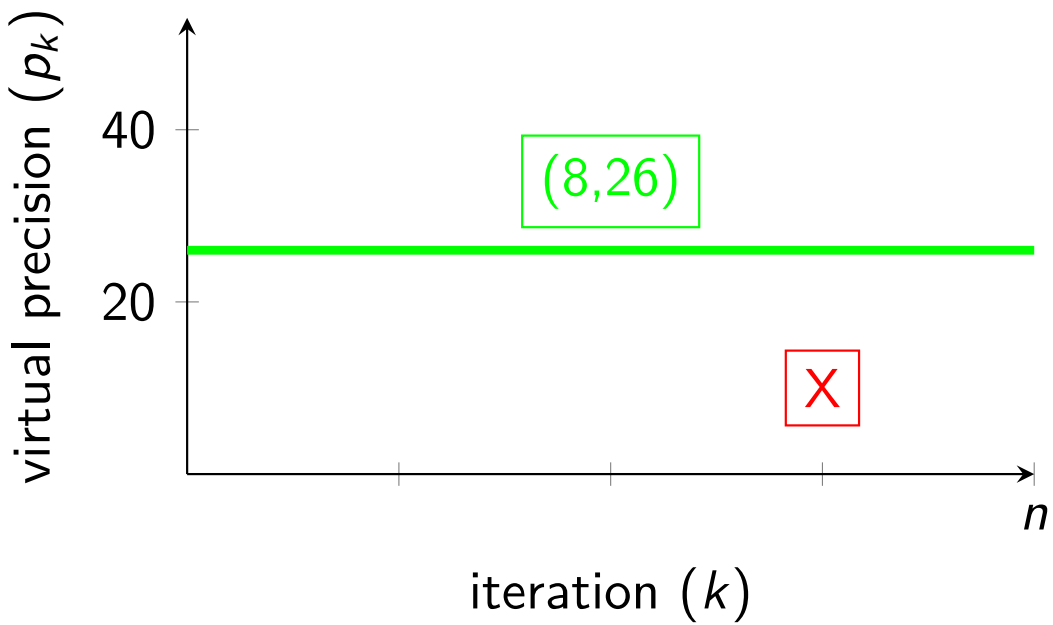
Lowest precision exploration

Objective: Find lowest valid configuration by enforcing four principles:

- Avoid combinatorial explosion ($(rp)^n$ combinations)
- Avoid configurations that quickly oscillate
- Distribute the reduction among all iterations
- Reduce more the first iterations than last iterations

Valid configuration

- Configuration $c = [(r_0, p_0), ..., (r_n, p_n)]$
- Preserves accuracy and convergence



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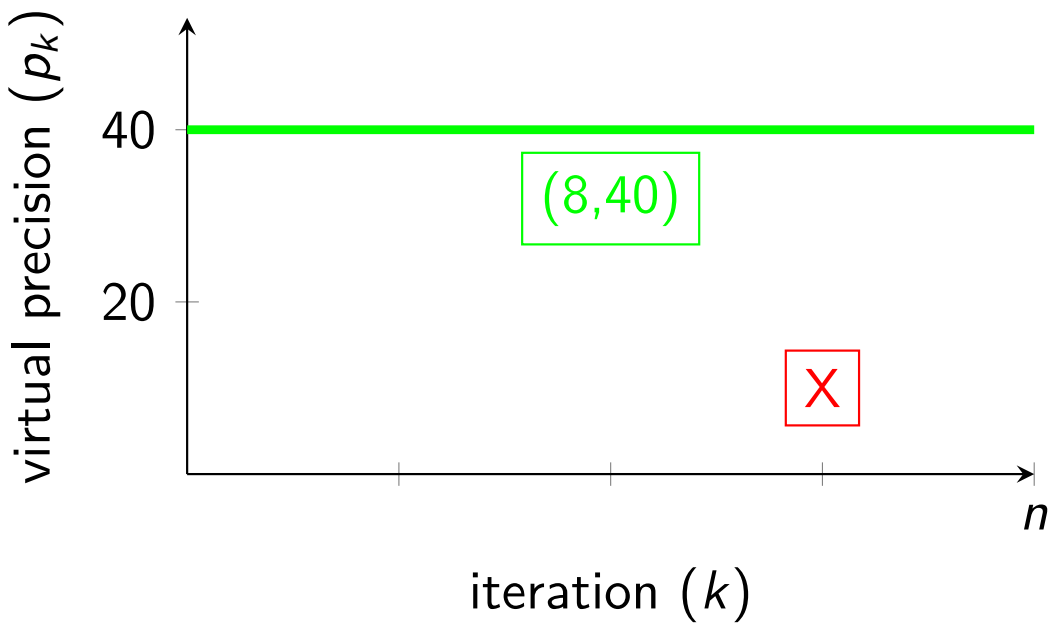

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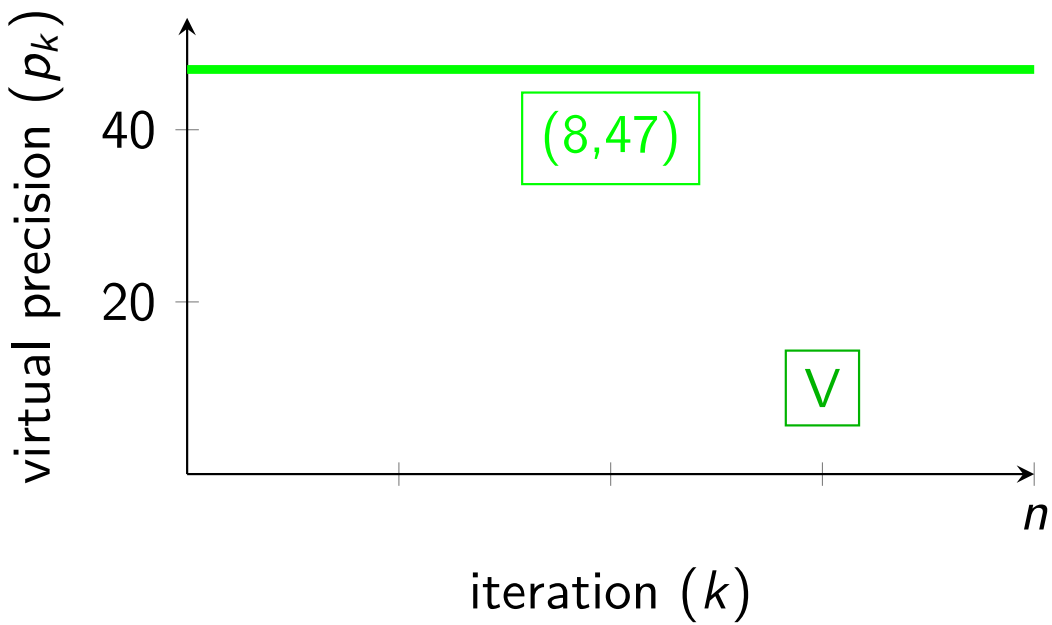

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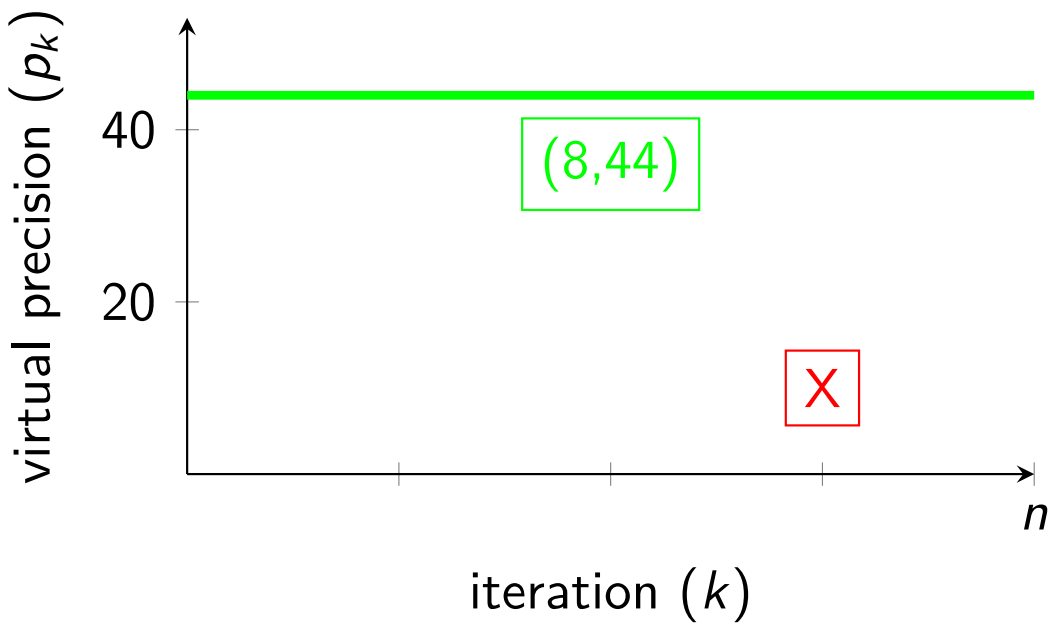

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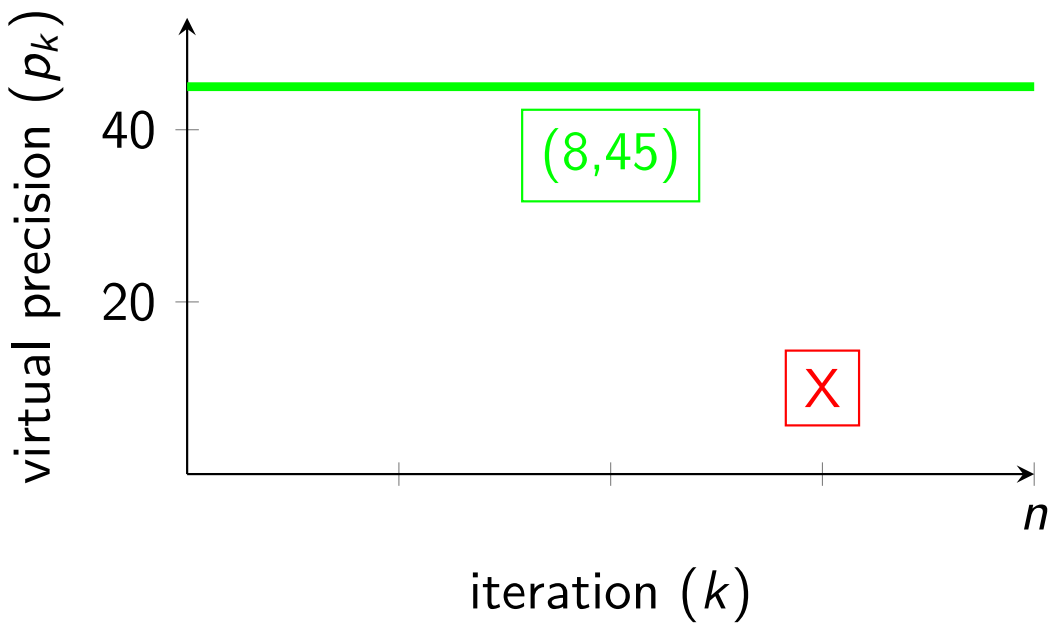

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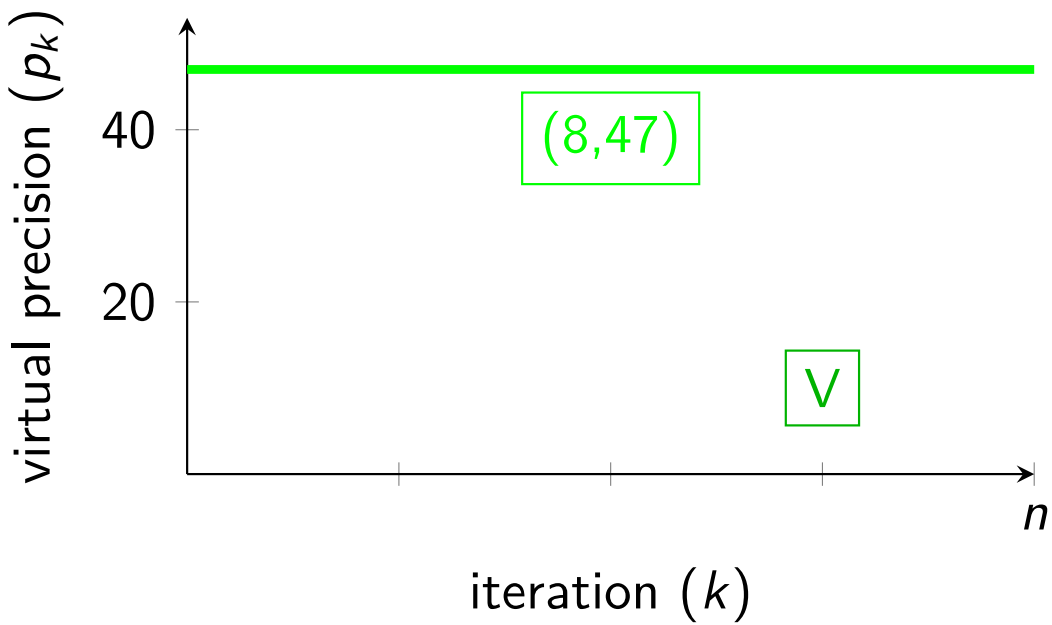

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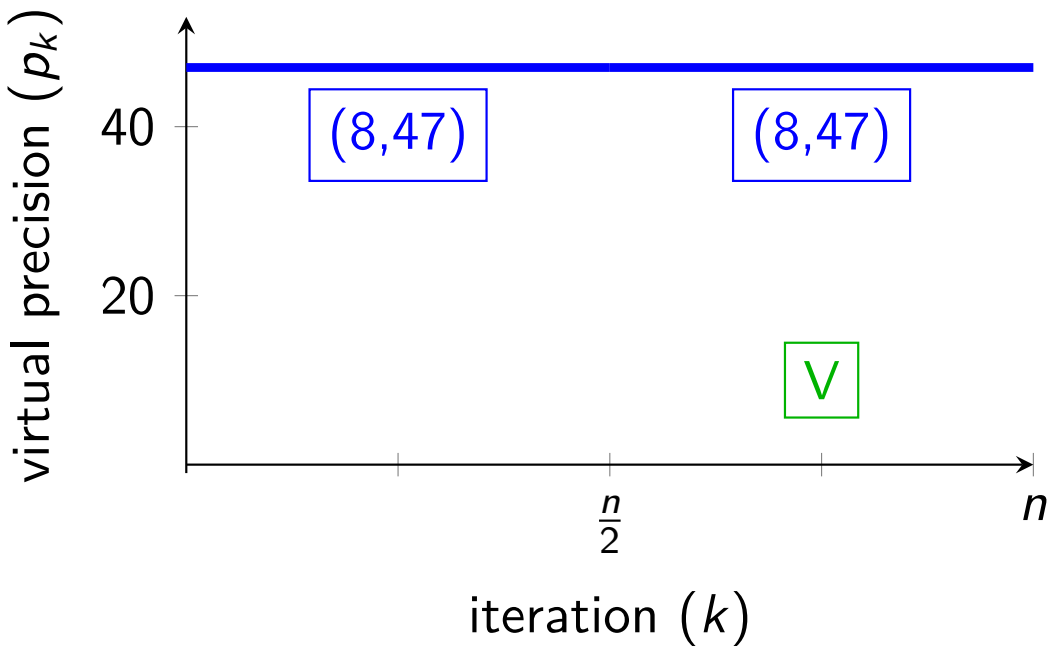

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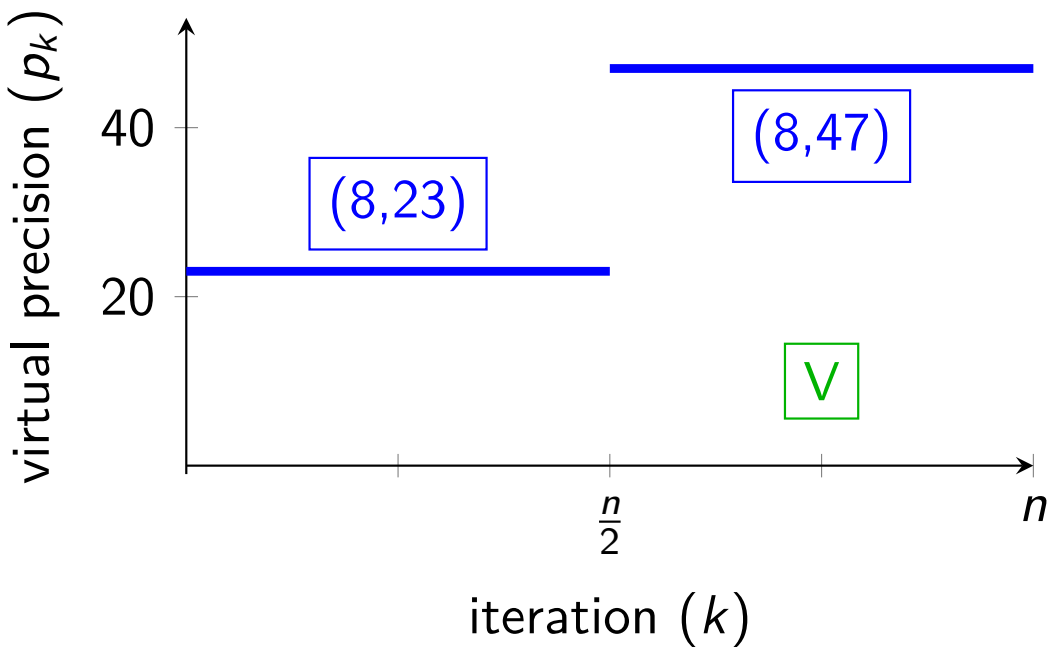

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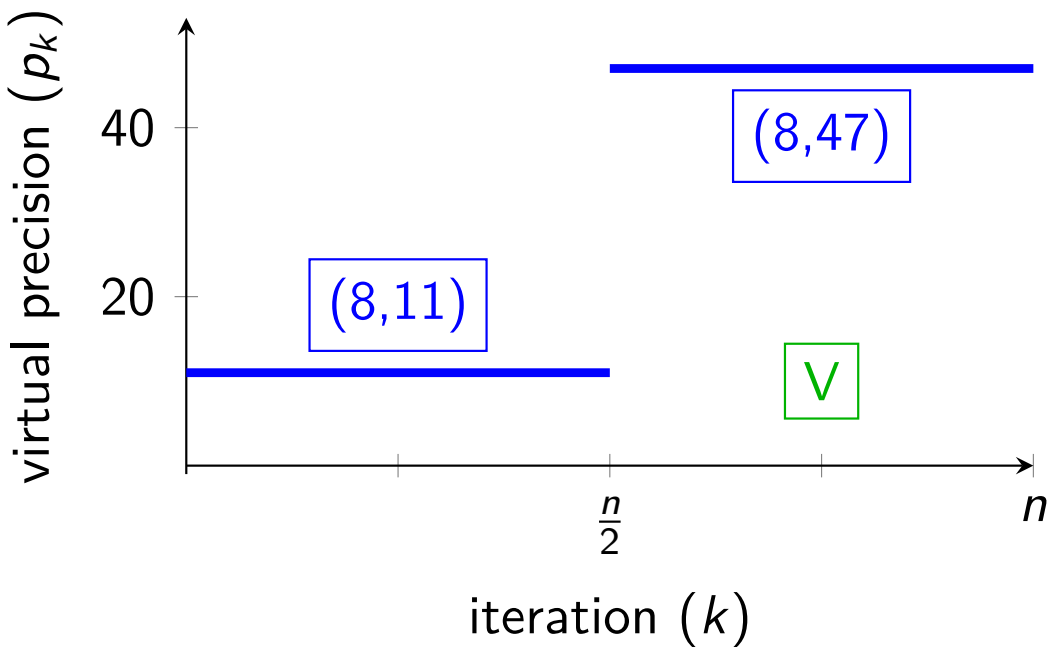

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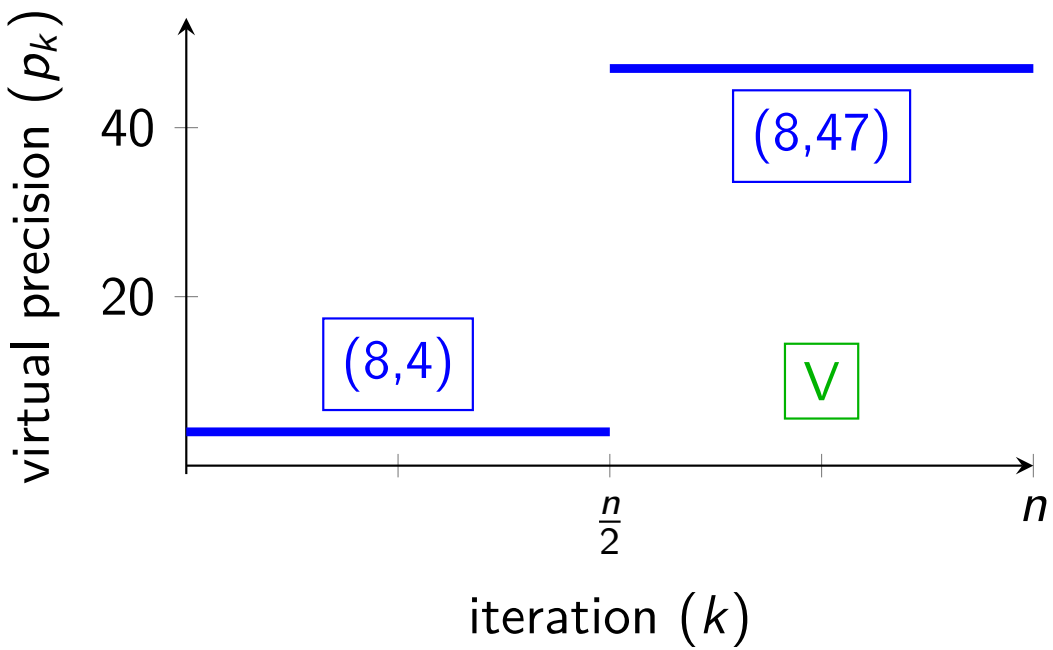

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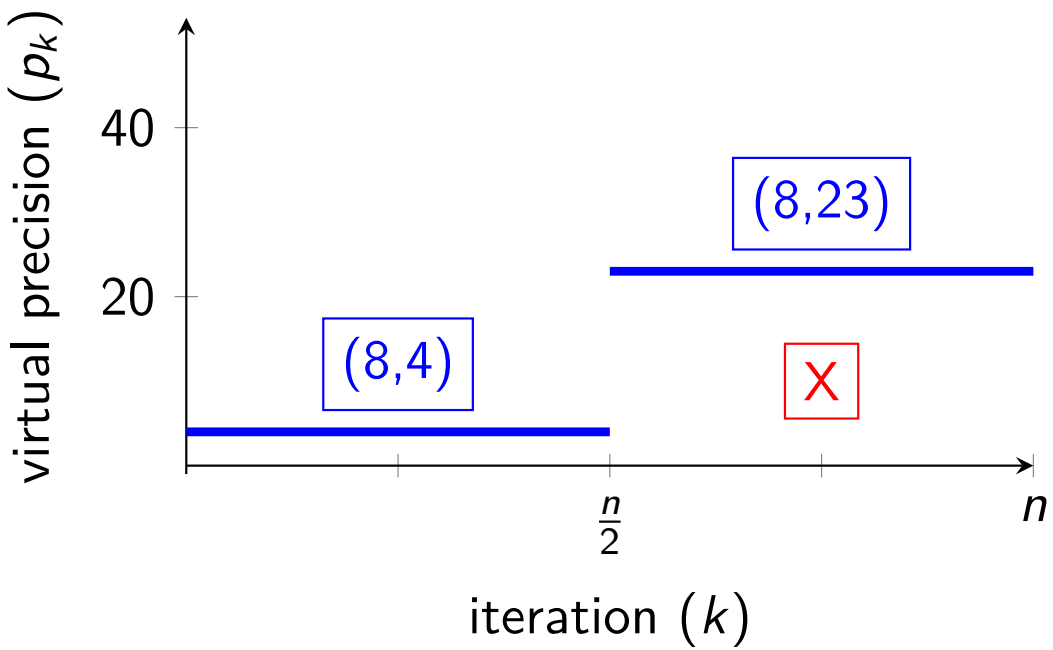

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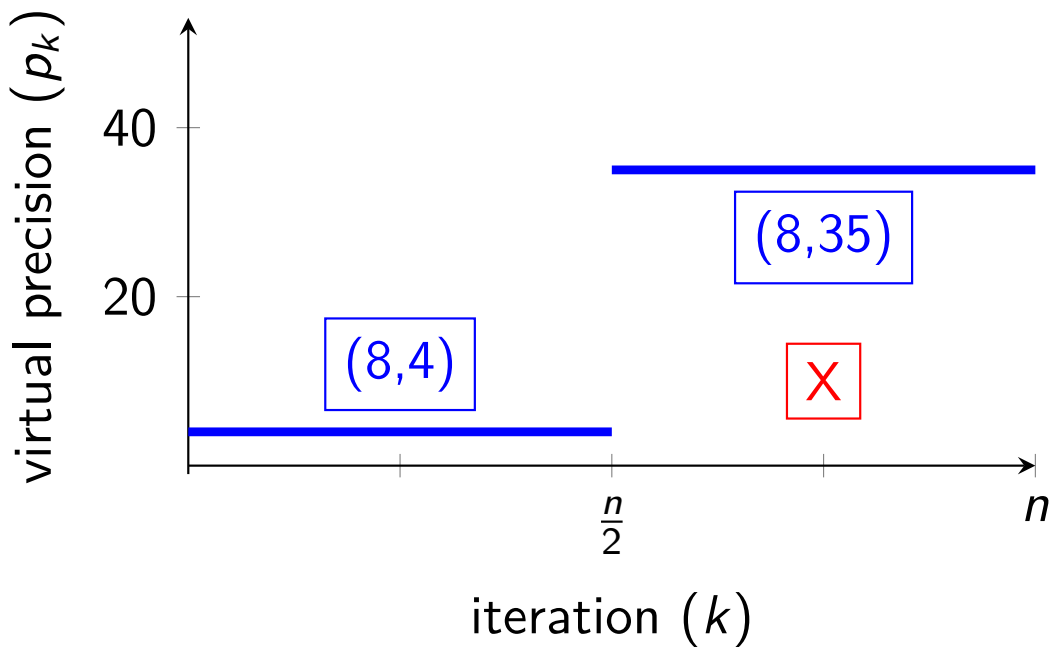

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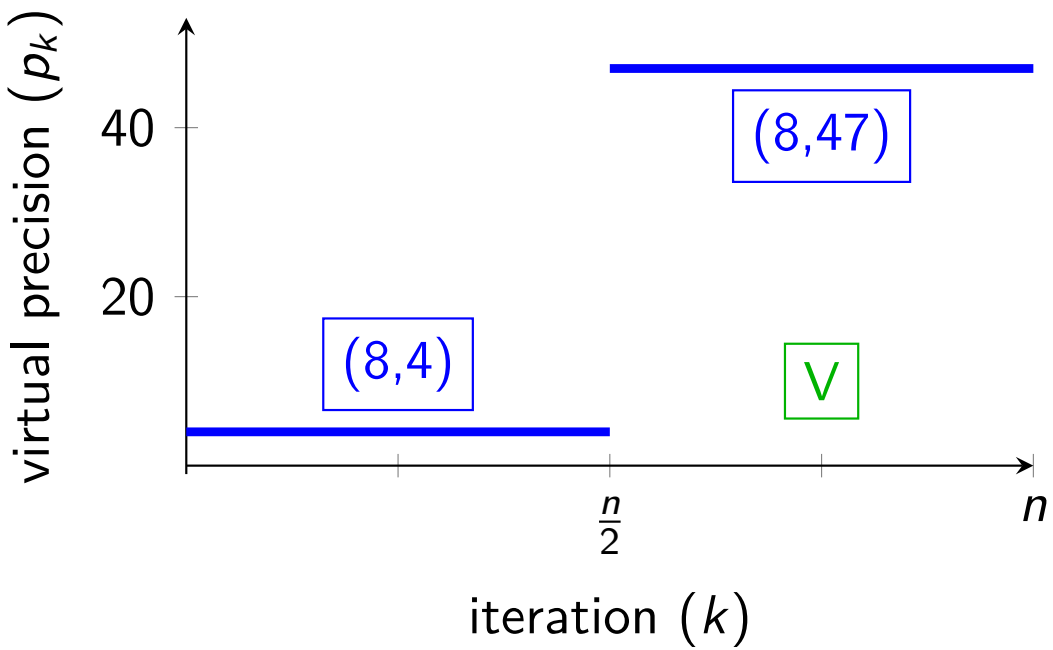

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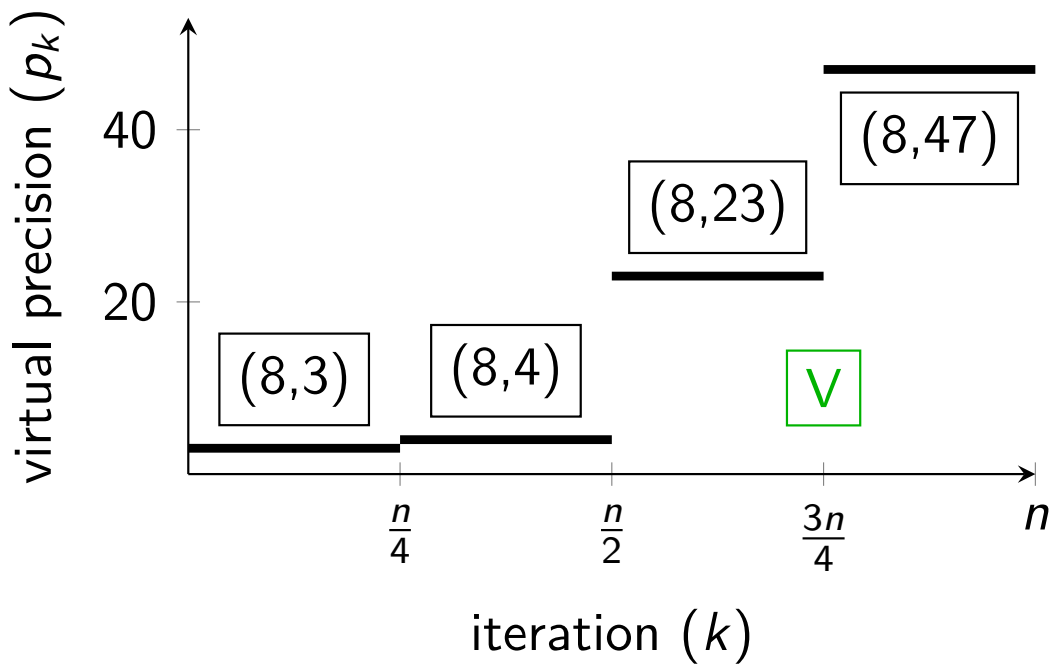

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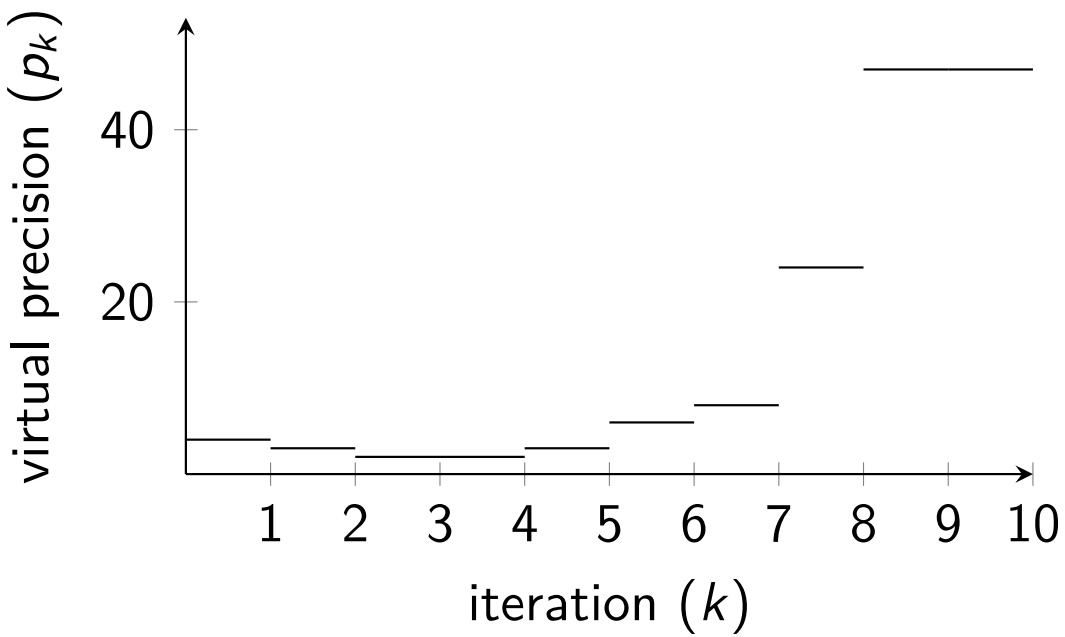


Exploration on an industrial use case: YALES2

Coria-CNRS and Safran, Solvay, GDF-Suez, www.coria-cfd.fr

Solver for two-phase combustion

- From primary atomization to pollutant prediction

Deflated Preconditioned Conjugate Gradient

- A fine grid is geometrically projected on a coarse grid
- A CG is applied on the coarse grid and on the fine grid

Preccinsta burner use case

- Evaluation on 1.75 million mesh elements
- Exploring reduction on *entire application* and *deflated grid*

Deflated Preconditioned Conjugate Gradient

A-DEF2 using preconditioner M^{-1} and deflation matrix W.

- In blue the coarse grid and green the fine grid
- deflated part = blue, entire application = green + blue

Require : A, b, M^{-1}, W
initialization
for k = 0 ,1 ,... until required convergence **do**

$$\alpha_{k+1} = \frac{r_k^T w_k}{p_k^T A p_k}$$

$$x_{k+1} = x_k + \alpha_{k+1} p_k$$

$$r_{k+1} = r_k - \alpha_{k+1} A p_k$$

Solve $\hat{A} d_{k+1} = W^T (A M^{-1} - I) r_{k+1}$

$$w_{k+1} = M^{-1} r_{k+1} - W d_{k+1}$$

$$\beta_{k+1} = \frac{r_k^T w_{k+1}}{r_k^T w_k}$$

$$p_{k+1} = w_{k+1} + \beta_{k+1} p_k$$

end for

Piecewise constant exploration on *preccinsta 1.75M*

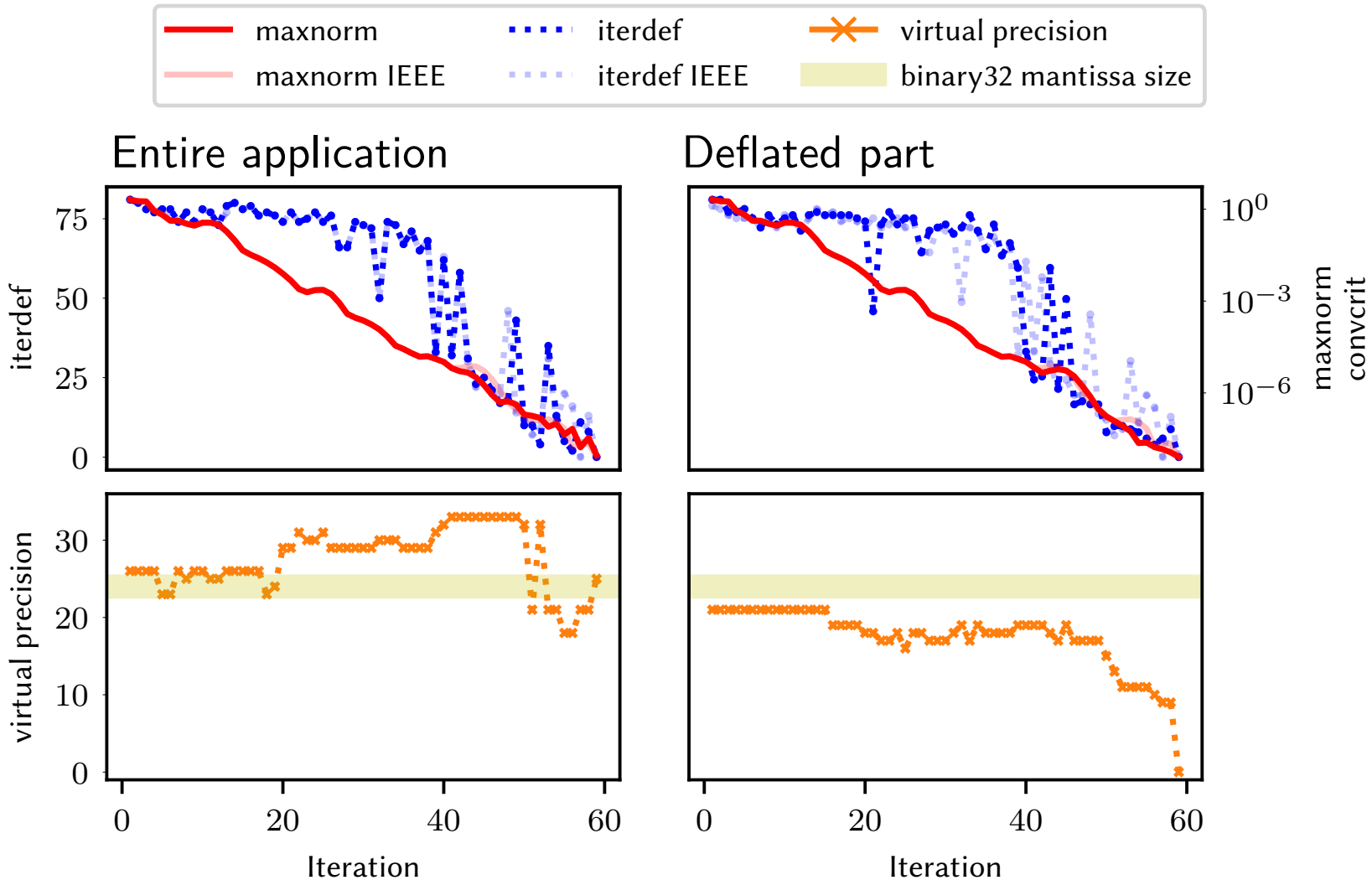


Figure: Adaptive precision searching on YALES2's DPCG on 1 MPI. $\mathbf{r} = \mathbf{8}$. Reduced precision solution follows the reference IEEE convergence profile

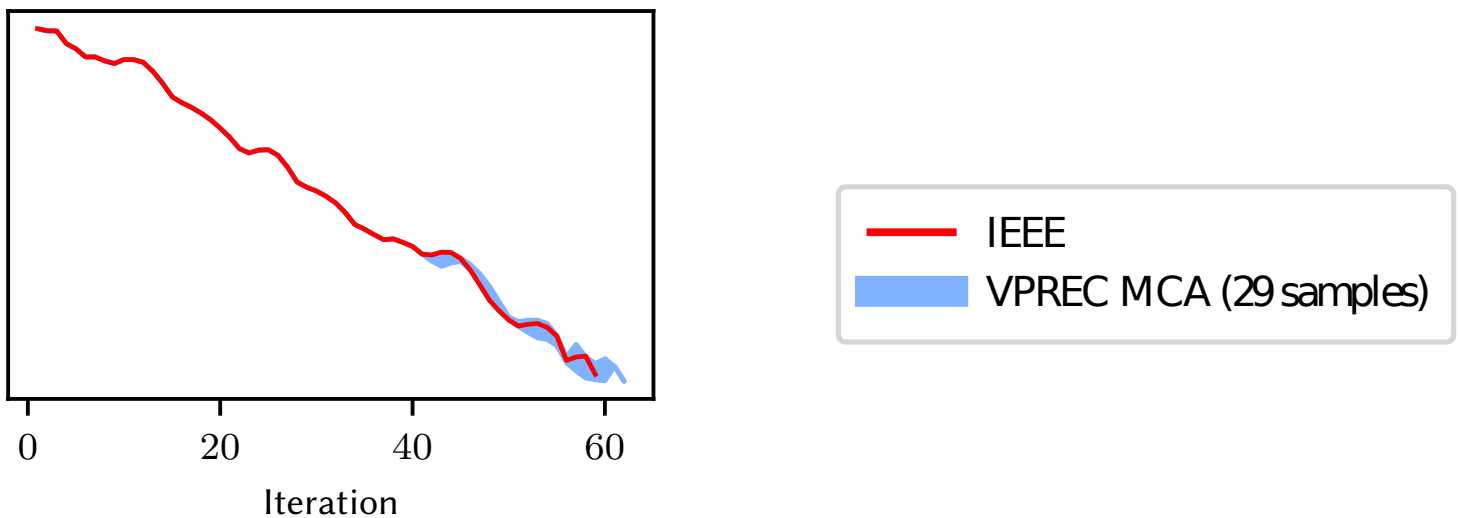
Validating resiliency to round-off errors

VPREC is a deterministic computation model

- How to validate configuration's robustness to rounding errors?

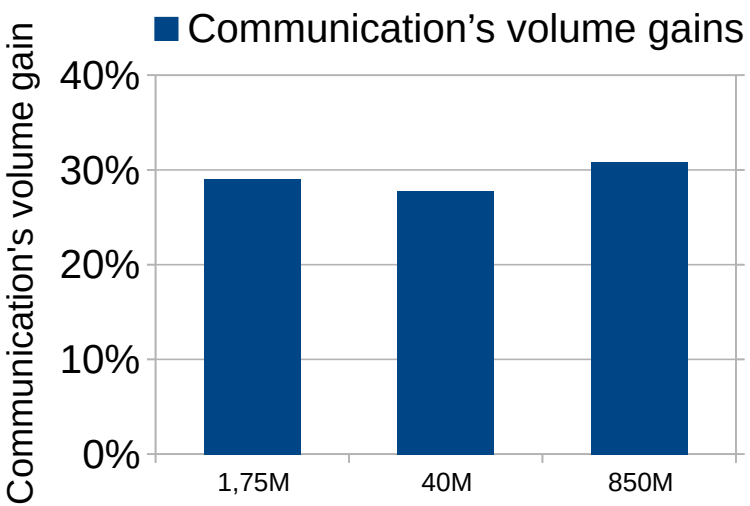
Using Monte Carlo Arithmetic [Parker, 1997]

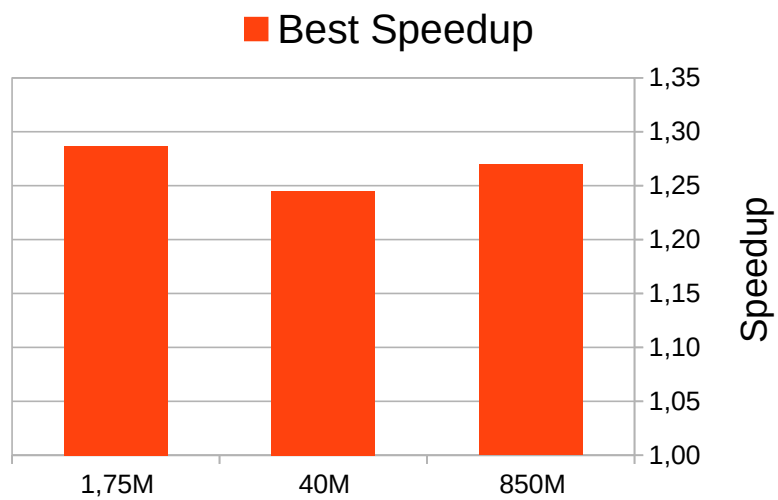
- Introduces random noise to model round-off errors at each FP ops
- Used as second step for validating VPREC configurations
- 29 converged MCA samples gives 90% of probability with 0.95 confidence level based on Bernouilli trials [Sohier et al., 2018] that configuration is robust to round-off errors



Performance validation on mixed-precision version

- Mixed-version to use both `binary32` and `binary64` formats in the deflated operator
- Switch from `binary32` to `binary64` when convergence criteria is below 10^{-7}
- Run up to 560 cores on CRIANN cluster (66 bisocket Intel Xeon E5-2680 nodes and Intel Omnipath interconnect)





Conclusion

VPREC

- Easily tests custom formats on HPC codes

Find lowest precision

- Automatic search to find lowest precision
- Reduces working precision while preserving accuracy and convergence
- Robustness to rounding errors thanks to MCA

Validation on industrial code Yales2

- Mixed-precision version scaling up to 560 cores
- Average communications' volume reduction by **30%** and best speedup of **1.28**

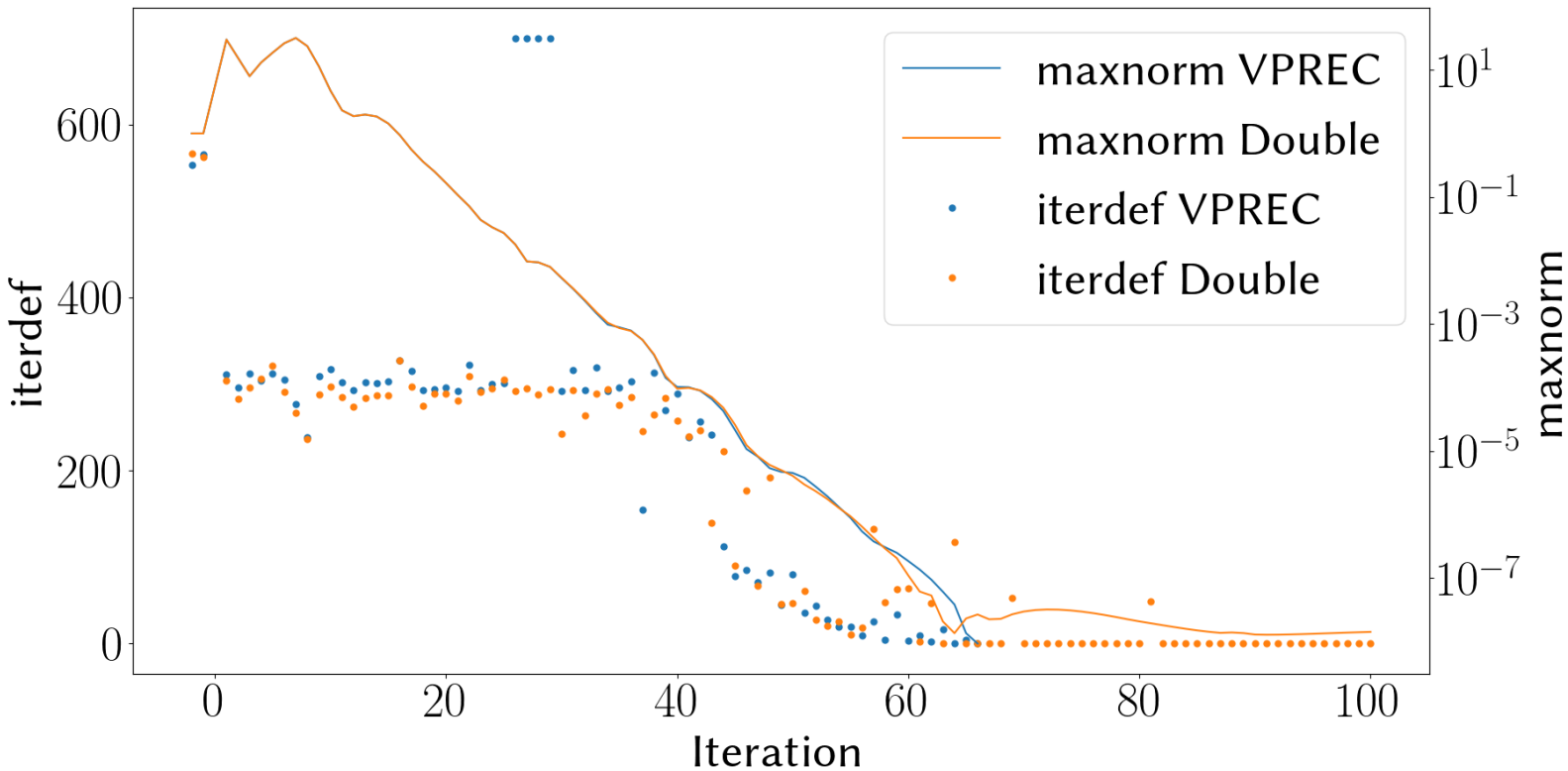
Backup slides

IEEE-754 formats

Name	Exponent (bits)	Pseudo-Mantissa (bits)	e_{min}, e_{max}
binary16	5	10	(-14,+15)
binary32	8	23	(-126,+127)
binary64	11	52	(-1022,+1023)
binary128	15	112	(-16382,+16383)

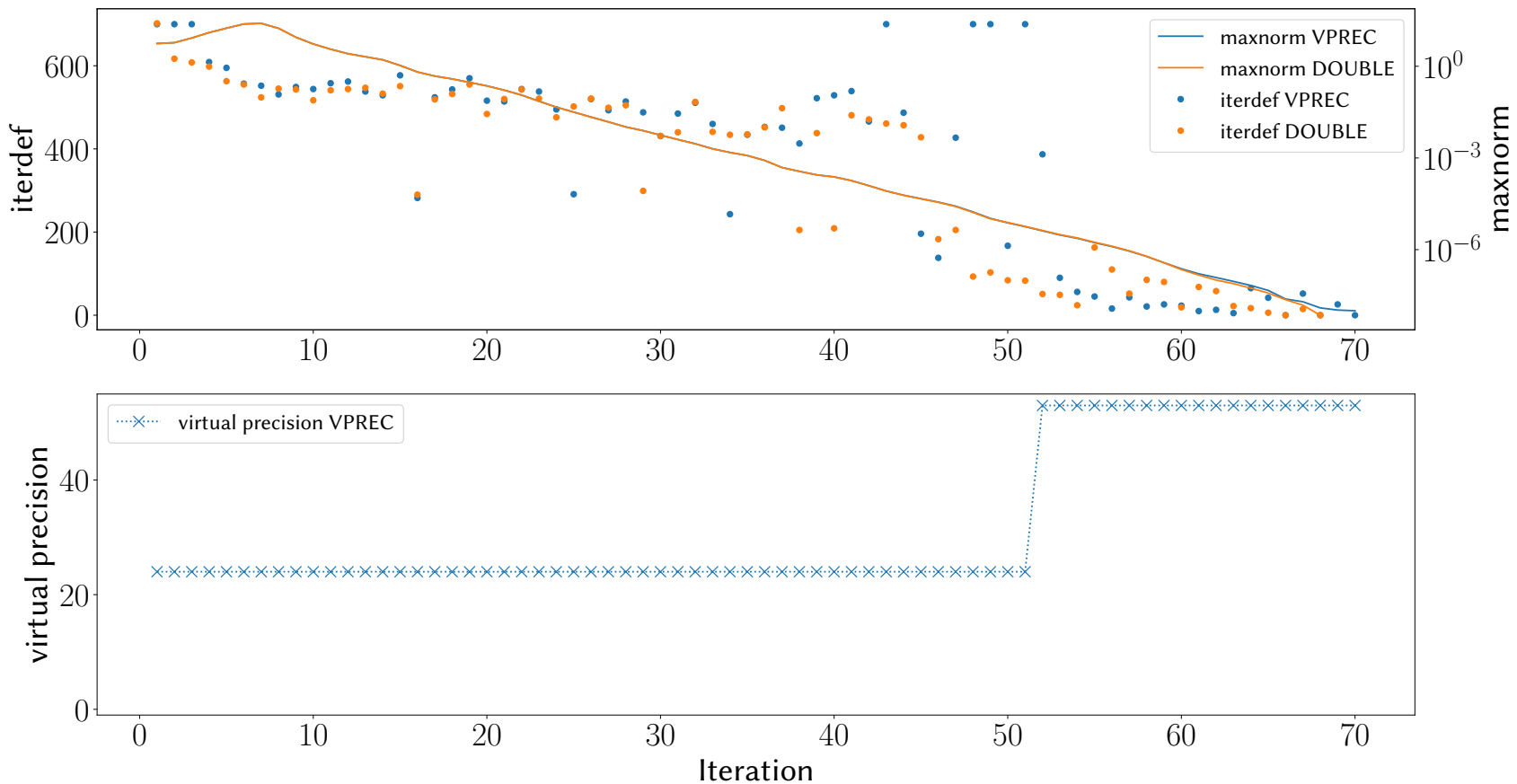
Table: Binary formats of the IEEE-754 norme

Accelerate the convergence



Exploration on *preccinsta* 850M on 112 MPI cores on deflated part. Solution found by the exploration (*VPREC*) converges in 66 iterations while the original (*Double*) converges in 100 iterations.
Reducing precision can be seen as a high-pass filter.
75% of solutions found by VPREC used less iterations than the IEEE version

Billion mesh elements



Exploration on *preccinsta* with billion of elements on 336 MPI cores. VPREC exploration shows that we need more precision at the end to reach the convergence. Fix $r = 8$

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